<u>Section 12.1-12.4:</u>
Vector Review

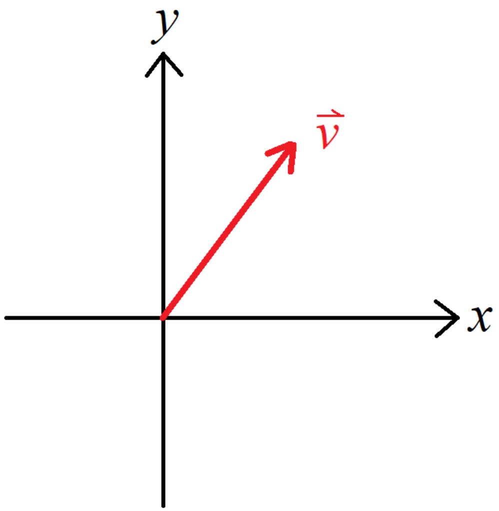
Vectors and Scalars

<u>Def</u>:

1. A <u>scalar</u> is a quantity that can be described by a single number.

2. A <u>vector</u> is a quantity that needs a number and a direction in order to describe it.

Visualizing Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$



Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$ are drawn as arrows

- The length of the arrow is the number part of the vector and is called its <u>magnitude</u>
- The direction the arrow is pointed in gives the <u>direction</u> of the vector
- The starting point doesn't matter, but usually vectors are drawn starting at the origin
- Even though in some classes, vectors are described with a magnitude and direction, in this class we will describe it by its endpoint (thinking that the starting point is always the origin)

Vector Notation

The notation for vectors in $\mathbb{R}^2$ look like…

$$< 2, -1 >$$

The notation for the set (group) of all vectors in $\mathbb{R}^2$ is…

$$V_2$$

Vector Notation

The notation for vectors in $\mathbb{R}^3$ look like…

$$< 2, -1, 5 >$$

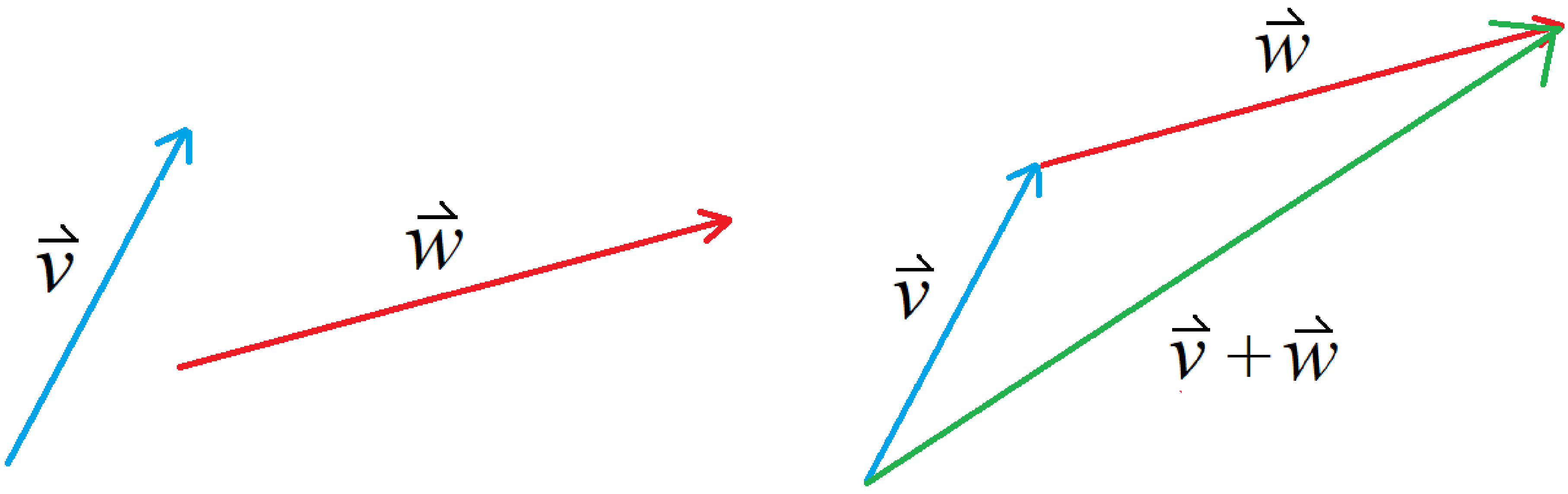
The notation for the set (group) of all vectors in $\mathbb{R}^3$ is…

$$V_3$$

Vector Operations: Addition

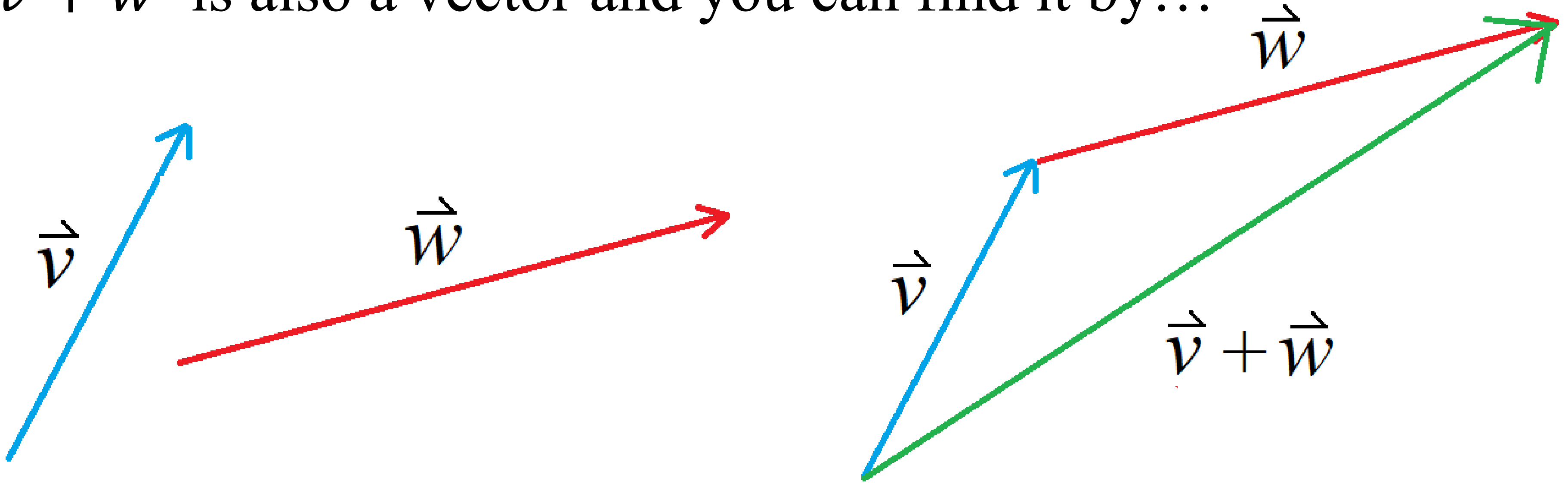
Visual: If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^2$ (or $\mathbb{R}^3$), their sum $\vec{v}+\vec{w}$ is also a vector and you can find it by…

1) Draw vector $\vec{v}$ first
2) Draw vector $\vec{w}$ in such a way that its tail is at the head of $\vec{v}$ (head-to-tail)
3) The vector $\vec{v}+\vec{w}$ is the vector whose tail is at the tail of $\vec{v}$ and whose head is at the head of $\vec{w}$



Vector Operations: Addition

<u>Visual</u>: If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^2$ (or $\mathbb{R}^3$), their sum $\vec{v}+\vec{w}$ is also a vector and you can find it by…



Adding vectors geometrically this way is equivalent to algebraically adding corresponding components together

Vector Operations: Addition

Adding vectors geometrically this way is equivalent to algebraically adding corresponding components together

Illustrate on board: Like $< 2, 5 > + < 3, -2 >$

Vector Operations: Addition

The way we'll add vectors…

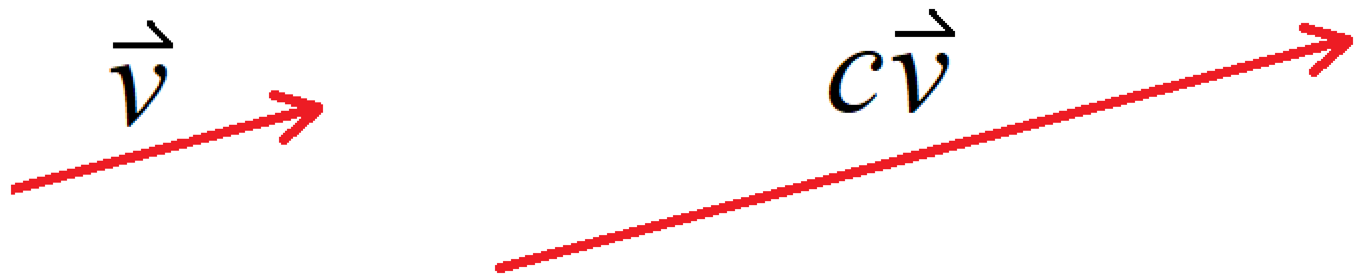
Note:

- To add vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$, add corresponding components together

Vector Operations: Scalar Multiplication

Visual:

If $\vec{v}$ is a vector in $\mathbb{R}^2$ (or $\mathbb{R}^3$) and c is a real number (scalar), you can multiply c and $\vec{v}$ to get a vector $c\vec{v}$ and you can find it by stretching vector $\vec{v}$ by a factor of c without changing its direction.



Vector Operations: Scalar Multiplication

Visual: More specifically…

If $c > 0$

the length of $c\vec{v}$ is the length of $\vec{v}$ multiplied by c

the direction of $c\vec{v}$ is in the same direction as $\vec{v}$

$c > 1$

$c = 1$

$0 < c < 1$

Vector Operations: Scalar Multiplication

Visual: More specifically…

If $c < 0$

the length of $c\vec{v}$ is the length of $\vec{v}$ multiplied by $|c|$

the direction of $c\vec{v}$ is in the opposite direction as $\vec{v}$

$c < -1$

$c = -1$

$-1 < c < 0$

Vector Operations: Scalar Multiplication

Visual: More specifically…

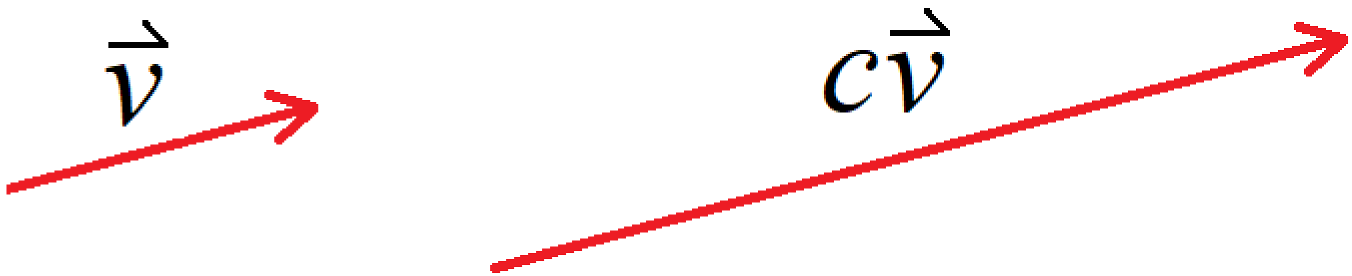
If $c = 0$

$c\vec{v}$ will just be the zero vector $\vec{0}$ which is just a dot at the origin. Its length is 0 and it doesn't have a direction (or it has all directions)

$c = 0$

Vector Operations: Scalar Multiplication

<u>Visual</u>:



Multiplying a vector by a scalar this way is equivalent to multiplying each component of $\vec{v}$ by the scalar c

Vector Operations: Scalar Multiplication

Multiplying a vector by a scalar this way is equivalent to multiplying each component of $\vec{v}$ by the scalar c

Illustrate on board: Like $2 < 1, 3 >$

Vector Operations: Scalar Multiplication

The way we'll do scalar multiplication…

Notes:

- To multiply a vector in $\mathbb{R}^2$ or $\mathbb{R}^3$ by a scalar, multiply each component by that scalar

Vector Operations: The Negative of a Vector

<u>Visual</u>:

If $\vec{v}$ is a vector in $\mathbb{R}^2$ (or $\mathbb{R}^3$), the negative of it has notation $-\vec{v}$ and can be visualized as an arrow with the same length as $\vec{v}$ but pointing in the opposite direction as $\vec{v}$.

Finding the negative of a vector this way is equivalent to changing the sign of each component of the vector.

Illustrate on board like $-< 2, -5 >$

Vector Operations: The Negative of a Vector

The way we'll find the negative of a vector…

Notes:

- To find the negative of a vector, change the sign of each component of the vector

Vector Operations

Ex 1: If $\vec{v} =< 2, -3, 7 >$ and $\overrightarrow{w} =< -6,1, 4 >$, find…

a) $\vec{v} + \overrightarrow{w}$

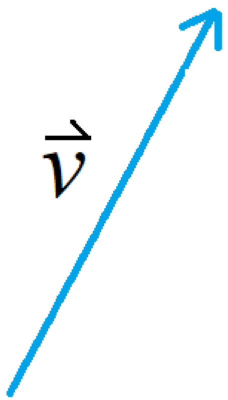
b) $-5\vec{v}$

c) $-\overrightarrow{w}$

Magnitude of a Vector

Visual:

If $\vec{v}$ is a vector in $\mathbb{R}^2$ (or $\mathbb{R}^3$) , its magnitude is denoted by $||\vec{v}||$ and geometrically represents the length of the arrow.



$||\vec{v}||$ is its length, like 3ft

Magnitude of a Vector

The way we'll find the magnitude of a vector…

<u>Def</u>:

1. If $\vec{v} = < a, b >$ is a vector in $\mathbb{R}^2$, then…

$$\|\vec{v}\| \equiv \sqrt{a^2 + b^2}$$

2. If $\vec{v} = < a, b, c >$ is a vector in $\mathbb{R}^3$, then…

$$\|\vec{v}\| \equiv \sqrt{a^2 + b^2 + c^2}$$

Magnitude of a Vector

Ex 2: Find the magnitude of $< -2, 5, 1 >$

Vector Operations: Dot Product

Given 2 vectors in $\mathbb{R}^2$ (or 2 vectors in $\mathbb{R}^3$), there are many ways to multiply them. One way is called the <u>dot product</u>.

Things to know about the dot product

- In order to find the dot product of 2 vectors, they must have the same number of components.
- The dot product of 2 vectors is a scalar.
- The dot product has properties that look like multiplication properties.
- Main uses of dot product is in finding the angle between 2 vectors and gives another way to find the magnitude of a vector.

Vector Operations: Dot Product

The way we'll find the dot product…

<u>Def</u>:

1. If $\vec{v} = < a, b >$ and $\vec{w} = < c, d >$ are vectors in $\mathbb{R}^2$, then…

$$\vec{v} \cdot \vec{w} \equiv ac + bd$$

2. If $\vec{v} = < a, b, c >$ and $\vec{w} = < d, e, f >$ are vectors in $\mathbb{R}^3$, then…

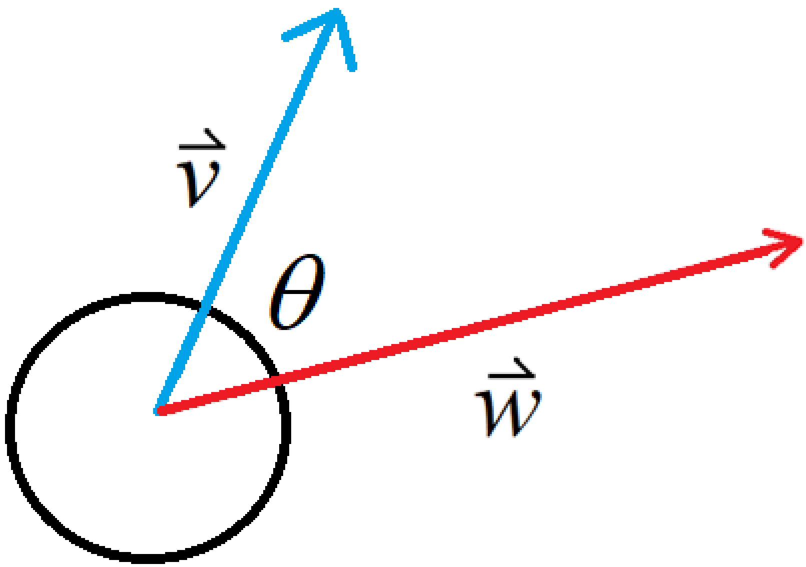
$$\vec{v} \cdot \vec{w} \equiv ad + be + cf$$

Vector Operations: Dot Product

Ex 3: Find $< 2,5,-3 > \cdot < 6,-4,0 >$

Vector Operations: Dot Product

Def: The angle between 2 vectors $\vec{v}$ and $\vec{w}$ is the smaller of the 2 angles when vectors $\vec{v}$ and $\vec{w}$ are drawn tail-to-tail.



Vector Operations: Dot Product

Theorem: If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^2$ (or $\mathbb{R}^3$) and θ is the angle between $\vec{v}$ and $\vec{w}$, then…

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos\theta \qquad \text{or} \qquad \theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}\right)$$

Vector Operations: Dot Product

Ex 4: Find the angle between the vectors

$$\vec{v} =< -1, 4, 2 > \text{ and } \overrightarrow{w} =< 7, -2\ \ -2 >$$

Vector Operations: Dot Product

Corollary: Let $\vec{v}$ and $\overrightarrow{w}$ be non-zero vectors in $\mathbb{R}^2$ (or $\mathbb{R}^3$). Then $\vec{v}$ and $\overrightarrow{w}$ are orthogonal (or perpendicular) if and only if $\vec{v} \cdot \overrightarrow{w} = 0$.

Vector Operations: Dot Product

Ex 5: Are the following pairs of vectors orthogonal?

a) $\vec{v} = < -1, -5 >$ and $\vec{w} = < 3, -2 >$

b) $\vec{v} = < -3, -2, 2 >$ and $\vec{w} = < 2, -1, 2 >$

Vector Operations: Cross Product

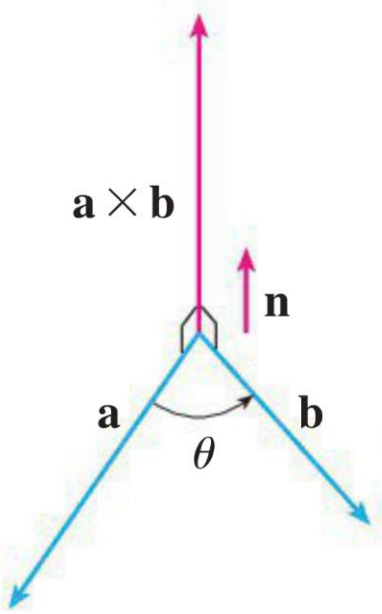
Given 2 vectors in $\mathbb{R}^3$, there is another way to multiply them called the cross product.

Things to know about the cross product

- In order to find the cross product of 2 vectors, they must be vectors in $\mathbb{R}^3$
- The cross product of 2 vectors is a vector.
- The cross product has properties that look like multiplication properties.
- The main use of the cross product is in finding a vector perpendicular to 2 given vectors.

Vector Operations: Cross Product

8 Theorem The vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.



Vector Operations: Cross Product

Ex 6: If $\vec{v} = < 4, -2, 1 >$ and $\vec{w} = < 3, 1, 5 >$, find $\vec{v} \times \vec{w}$.

Position Vectors

Notes:

- The starting of a position vector MUST BE THE ORIGIN
- We usually use $\vec{r}$ (sometimes $\vec{x}$) for position vectors
- A position vector gives you the position (location) of a point from the origin

Position Vectors

<u>Notes:</u>

$$\text{point} = (a, b, c) \qquad \leftrightarrow \qquad \text{position vector} = < a, b, c >$$

This allows us to do algebra with points

Vector from point P to point Q

If P $= (a, b, c)$ and Q $= (d, e, f)$, then

$$\overrightarrow{PQ} =< d - a, e - b, f - c >$$

Ex 7: Find the vector $\vec{v}$ whose tail is at point $(3, -5, 2)$ and whose head is at point $(0,3, -1)$